a) $y = x^{e^x}$ e^x x^{e^x-1} −7 pts

$\ln y = e^x \ln x$

$\frac{1}{y}\frac{dy}{dx} = e^x \ln x + \frac{e^x}{x}$

$\frac{dy}{dx} = x^{e^x}\left(e^x(\ln x + \frac{1}{x})\right)$

$= x^{e^x} e^x \left(\frac{x\ln x + 1}{x}\right)$

b) $y = x^{x^e}$

$\ln y = x^e \ln x$

$\frac{1}{y}\frac{dy}{dx} = ex^{e-1}\ln x + \frac{e^x}{x}$

$\frac{dy}{dx} = \left(\frac{e\,x^e \ln x + e^x}{x}\right) x^{x^e}$

$\frac{e(x^e \ln x + e^{e-1}) x^{x^e}}{x}$

c) $y = e^{x^e}$

$\frac{dy}{dx} = e^{x^e} \cdot e x^{e-1}$

$= e^{x^e+1} x^{e-1}$

d) $y = e^{e^e}$

$\frac{dy}{dx} = 0$

y is a constant

e) $y = e^{e^x}$

$\frac{dy}{dx} = e^{e^x} \cdot e^x$

$= e^{e^x + x}$

f) $y = \log_{10} x^e$

$y = \frac{e \ln x}{\ln 10}$

$\frac{dy}{dx} = \frac{e}{x \ln 10}$

g) $y = \log_{10} e^x$

$y = \frac{x \ln e}{\ln 10}$

$\frac{dy}{dx} = \frac{\ln e}{\ln 10}$

II $f(x) = \overset{u}{x}\overset{v}{e^x}$ $x \in [-3, e]$

$f'(x) = e^x + xe^x$ $\quad f' \; - \; 0 \; + \;$ (−3, −1, 0, e)

$e^x(1+x)$

$f''(x) = e^x + e^x + xe^x$ $\quad f'' \; - \; 0 \; + \;$ (−2, 0)

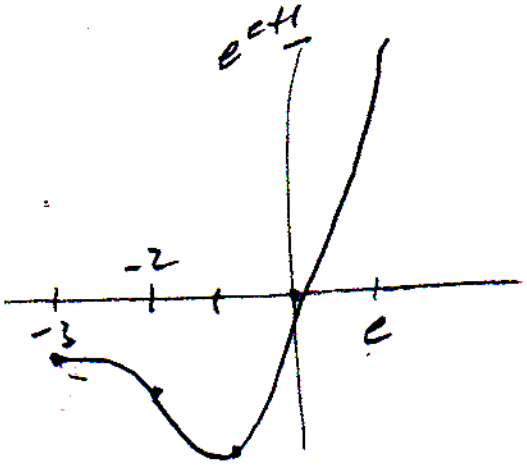
$= e^x(2+x)$

$(-3, \frac{-3}{e^3})$ max & WPT dec fn'y.

$(-2, \frac{-2}{e^2})$ PI changes down to up

$(-1, \frac{-1}{e})$ min f' (not)

(e, e^{e+1}) max inc to f.

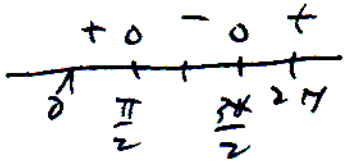


III $f(x) = \cos x \quad 0 \le x \le 2\pi$

$g(x) = \ln x \quad x > 0$

$S(x) = g(f(x))$

a) $D_S \quad \cos x > 0$



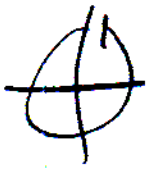
$D_S \quad 0 \le x < \frac{\pi}{2}$ or $\frac{3\pi}{2} < x \le 2\pi$

b) $R_S \quad \ln(\star) <$ if $\star < 1$ cosine is < 1 $\therefore R_S \; S(x) \le 0$

c) $S = \{0\}$ $\quad \ln \star = 0$ if $\star = 1$ $\quad \cos x = 1$, $x \in \{0, 2\pi\}$

D) $S'(x) =$

$S = \ln(\cos x)$



$S' = -\frac{\sin x}{\cos x}$

$-\tan x \Big|_{x=\frac{\pi}{3}}$ $\quad -\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$

$= -\sqrt{3}$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

Extra

$\int_0^2 (2x-1)^4 \, dx$

$(2x)^4 + 4(2x)^3 + 6(2x)^2 - 4(2x) + 1$

$\int 16x^4 - 32x^3 + 24x^2 - 8x + 1$

$\frac{16x^5}{5} - 8x^4 + 8x^3 - 4x^2 + x \Big|_0^2$

$\frac{16 \cdot 32}{5} - 8 \cdot 16 + 8 \cdot 8 - 4 \cdot 4 + 2$

$-8(16+8)$

$-64 - 16 + 2$

-78

$\frac{512 + 390}{5}$

$\frac{122}{5}$

24.4

256
2
512
390
122

6/2/2000

100	0	2	2
80	1	4	5
80	5	2	7
70	1	4	5
60	2	4	6
59	12	4	16
	12+2	14+8	26+8 25

100

62.1 75.6 67.0

most common error
was $(e^x)' = x e^{x-1}$
instead of $e^x dx$.

this error accounts for
about 15% on the overall
scores.
so what that are
grade would have been
over 80!

Honors Analysis

Name AP Ch 5, 6, 7

Per _____ Date 6/2/2000

Copy original problem.

Convince *me* that **you** understand the concept!

No Calculators!

95

Last Regular Exam

I Given y, find $\frac{dy}{dx}$.

Final answers are to be fully factored, single fractions (if any) and to have no negative exponents.

(60 pts tot)

a) $y = x^{e^x}$ b) $y = x^{x^e}$ c) $y = e^{x^e}$ d) $y = e^{e^e}$

e) $y = e^{e^x}$ f) $y = \log_{10} x^e$ g) $y = \log_{10} e^x$

II Determine the coordinates of significant points. Sketch. Justify.

$f(x) = x e^x; \quad D_f \quad x \in [-3, e]$ (15 pts)

III Given: 1) $f(x) = \cos x$ for $0 \le x \le 2\pi$

2) $g(x) = \ln x$ for $x > 0$

3) $S(x) = g(f(x))$ (25 pts tot)

a) Find the Domain of $S(x)$.

b) Find the Range of $S(x)$.

c) Find the zeros of $S(x)$.

d) Find the slope of the line tangent to the graph of $S(x)$ at $x = \frac{\pi}{3}$

Extra Credit ---------- 5 pts ----------

Evaluate: $\int_0^2 (2x-1)^4 \, dx$

I $f(x) = \frac{x}{\ln x}$ (u / v)

a) $\ln x$: − | 0 | +; x: − | 0 | +

$x > 0,\ x \neq 1$ (3 of 3)

b) $f'(x) = \frac{\ln x - x\frac{1}{x}}{(\ln x)^2} = \frac{\ln x - 1}{(\ln x)^2}$

		0		1		e
u	−	‖	−	0	+	
v	+	0	+	‖	+	
f'	−	‖	−	0	+	

f is increasing $\forall x > e$

c) only extrema is a relative minimum at (e, e) by f' test.

d) $f''(x) = \frac{(\ln x)^2 \frac{1}{x} - (\ln x - 1)\frac{2\ln x}{x}}{(\ln x)^4}$

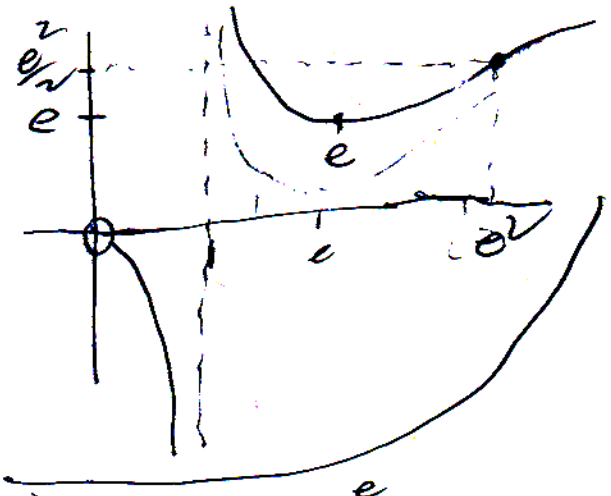
$= \frac{\frac{\ln x}{x} - \frac{2\ln x}{x} + \frac{2}{x}}{(\ln x)^3}$

$= \frac{2 - \ln x}{x(\ln x)^3} \overset{\text{set}}{=} 0$

		0		1		e		e^2
$2 - \ln x$	+		+		+	0	−	
x	−	‖	+		+		+	
$(\ln x)^3$	−	‖	−	0	+		+	
f''	−	‖	+		+	0	−	

f is concave down for $0 < x < 1$ or $x > e^2$

e) $R_{f(x)}$ $f(x) < 0$ or $f(x) \geq e$



II

1) $f(x) = \sin e^{-2x}$

$f'(x) = -2e^{-2x}\cos e^{-2x}$

f) $f(x) = \ln \tan x$

$= \ln(\sin x) - \ln(\cos x)$

$= \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}$ OR $\frac{1}{\tan x}\sec^2 x$ / $\cot x \sec^2 x$

$= \frac{\cos^2 x - \sin x}{\sin x \cos x}$

$= \csc x \sec x$

c) $f(x) = e^{x^e}$

$f'(x) = e^{x^e} e x^{e-1} = e^{x^e + 1} x^{e-1}$

d) $f(x) = \frac{1}{\ln x} = (\ln x)^{-1}$

$f'(x) = -(\ln x)^{-2}\frac{1}{x}$

$= -\frac{1}{x(\ln x)^2}$

III

A) $\int \frac{(2x+3)^3}{x}\,dx$

$4x^2+12x+9$ / $2x+3$

$8x^3+24x^2+18x$

$12x^2+36x+27$

$\int 8x^2+36x+54+\frac{27}{x}\,dx$

$\frac{8x^3}{3}+18x^2+54x+27\ln|x|+C$

B) $\int e^{-kx}\,dx =$

$-\frac{1}{k}e^{-kx}+C$

IV

A) $\ln 40$

$\ln 2^3+\ln 5$

$3A+C$

B) $\log_{13}8 = \frac{3\ln 2}{\ln 3} = 3A$

$= 13^2$

C) $\ln(2.4)$

$\ln\left(\frac{12}{5}\right)$

$2\ln 2+\ln 3-\ln 5$

$2A+B-C$

D) $(\log_5 2)(\log_2 25)$

$\frac{\ln 2}{\ln 5}\cdot\frac{2\ln 5}{\ln 2}=2$

V

$e^y = x$

$\ln e^y = \ln x$

$y\ln e = \ln x$

$y = \ln x$

$\frac{dy}{dx} = \frac{1}{x}$

OR

$e^y\frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{e^y}$

$= \frac{1}{x}$

VI

$m = e^2 \quad (x, \ln x)$

$\frac{1}{x} \overset{\text{Set}}{=} e^2 \quad (e^{-2}, -2)$

$\therefore x = e^{-2}$

$y+2 = e^2(x-e^{-2})$

$y+2 = e^2x-1$

$y = e^2x-3 \overset{\text{Set}}{=} 0$

$x = \frac{3}{e^2}$

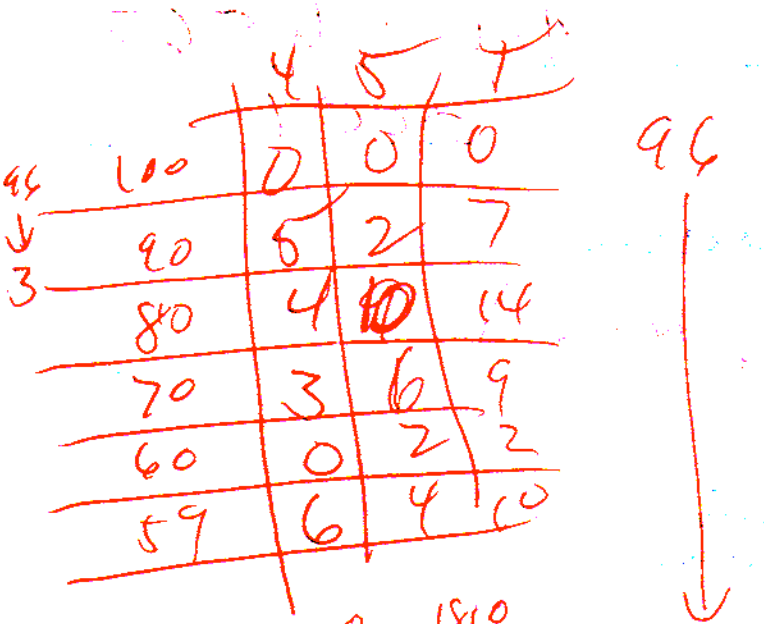
Extra.

$f(x) = \ln(\cos^2x-\sin^2x)$

$= \ln(\cos 2x)$

$f'(x) = -\frac{2\sin 2x}{\cos 2x} = -2\tan 2x$

$f'\left(\frac{\pi}{8}\right) = -2\tan\frac{\pi}{4} = -2.$



Per 4 The bottom 4 Gave up a hill a 50.

This was an excellent exam. I could add to part I ~~the~~ on " given: $\lim_{x \to 0+} f(x) = 0$ "

Yes. I believe that would have good chance since they really can't do those limits themselves.

Honors Analysis

Name AnL Ch 5&7

Copy the original problem.
Convince *me* that **you** understand the concept.
No calculators, of course.

Per _____ Date 2001

LAST REGULAR EXAM

I Given: $f(x)=\dfrac{x}{\ln x}$ (hint $\lim_{x\to 0^+} f(x)=0$.) (tot 30 pts)

a) Find the domain of $f(x)$
b) Find the interval(s) where $f(x)$ is increasing.
c) Find and identify the *coordinates* of all extrema.
d) Find the interval(s) where $f(x)$ is concave down.
e) What is the range of $f(x)$.
f) Based on sections "a" through "d", sketch $f(x)$.

II Given $f(x)$, find $f'(x)$: (5 pts ea)

a) $f(x)=\sin e^{-2x}$
b) $f(x)=\ln(\tan x)$
c) $f(x)=e^{x^e}$
d) $f(x)=\dfrac{1}{\ln x}$

III Find the integral: (5 pts ea)

a) $\displaystyle\int \frac{(2x+3)^3}{x}\,dx$
b) $\displaystyle\int e^{-kx}\,dx$

IV Simplify using the "Mumaugh" tables: (5 pts ea)

a) $\ln 40$ b) $\log_3 8$ c) $\ln(2.4)$ d) $(\log_5 2)(\log_7 25)$

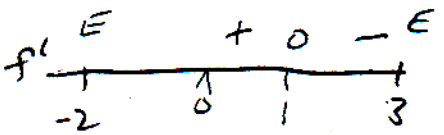
V Given $e^y = x$. Find the first derivative of y with respect to x. (10 pts)

VI Find the x-intercept of the line tangent to $f(x)=\ln x$ which has slope e^2. (10 pts)

Extra Credit 5 pts

Given: $f(x)=\ln(\cos^2 x-\sin^2 x)$, find $f'\left(\dfrac{\pi}{8}\right)$.

I $f(x) = \frac{x}{e^x}$ $f' = \frac{e^x - xe^x}{e^{2x}}$



a) $f'(x) = \frac{1-x}{e^x}$ $= \frac{1-x}{e^x}$

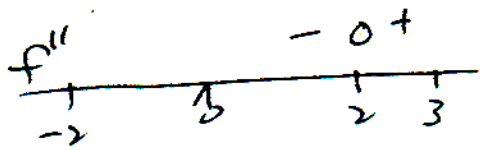
b) f increases $-2 \leq x < 1$

c) $(-2, -2e^2)$ min endpt inc fun't.
$(1, \frac{1}{e})$ max f'' test
$(3, \frac{3}{e^3})$ min endpt
dec to it.

$f'' = \frac{-e^x - e^x(1-x)}{e^{2x}}$

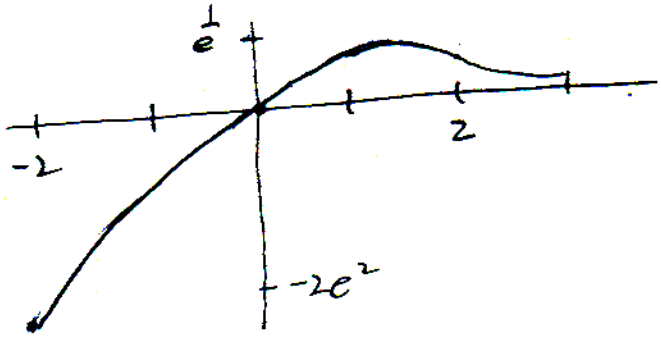
$\frac{-1 - 1 + x}{e^x}$

$\frac{-2+x}{e^x}$



d) f concave down
$-2 \leq x < 2$

e) R_f $-2e^2 < f(x) < \frac{1}{e}$



II a) $f(x) = e^{\cos x}$

$f'(x) = -\sin x\, e^{\cos x}$

b) $f(x) = x^{e^x}$ Ignore Not Linear −4

$\ln y = e^x \ln x$

$\frac{1}{y}\frac{dy}{dx} = e^x \ln x + \frac{e^x}{x}$

$f'(x) = x^{e^x}\left(e^x(\ln x + \frac{1}{x})\right)$

c) $f(x) = \ln e^{5x}$

$= 5x \ln e$

$= 5x$

$f'(x) = 5$

d) $f(x) = \ln\left(\frac{\sin x}{\cos x}\right)$

$= \ln \tan x$

$f'(x) = \frac{\sec^2 x}{\tan x} = \frac{1}{\cos^2 x} \cdot \frac{\cos x}{\sin x}$

$= \frac{1}{\sin x \cos x} = \sec x \csc x$

III $f(x) = e^x$

$f'(x) = e^x \overset{SET}{=} 2$

$\ln e^x = \ln 2$

$x = \ln 2$

$(x, e^x) \to (\ln 2, 2)$ $m = 2$

$y - 2 = 2(x - \ln 2)$

$y = 2x - 2\ln 2 + 2$

$0 \overset{SET}{=} 2x - 2\ln 2 + 2$

$\ln 2 - 1 = x$

x intercept

$(\ln 2 - 1, 0)$

IV

$$\int_{\frac{\pi}{3}}^{\frac{4\pi}{3}} \cos x + 2\,dx = \sin x + 2x \Big|_{\frac{\pi}{3}}^{\frac{4\pi}{3}}$$

$$-\frac{\sqrt{3}}{2} + \frac{8\pi}{3} - \left(\frac{\sqrt{3}}{2} + \frac{2\pi}{3}\right)$$

$$-\sqrt{3} + 2\pi$$

V

$$y = x^{\frac{1}{\ln x}}$$

Explain, only -6

$$\ln y = \ln x^{\frac{1}{\ln x}}$$

$$= \frac{1}{\ln x} \cdot \ln x$$

$$\ln y = 1$$

so $y = e$

$$\lim_{x \to \infty} \left(e = x^{\frac{1}{\ln x}}\right)$$ which leads us to

$$e = \infty^0$$

Extra.

The time could be written.

12:34:56.7.8.90

I was 46 yrs old.

5(3.1)02

	3	5	t
100	0	2	
90	2	3	
80	6	9	
70	9	6	
60	0	5	
59	1	3	

105

27

1419/18 2145/27

78.8 79.4 79.2

a quat ⊖ 0.4m.

Honors Analysis Name AnL Ch 5 C7

Copy the original problem. Per _____ Date 2002
Convince *me* that **you** understand the concept.
No calculators, of course.

LAST REGULAR EXAM

I Given: $f(x) = \dfrac{x}{e^x}$; $-2 \le x \le 3$ (tot 35 pts)

a) Find $f'(x)$ in fully simplified form.
b) Find the interval(s) where $f(x)$ is increasing.
c) Find and identify the *coordinates* of all extrema.
d) Find the interval(s) where $f(x)$ is concave down.
e) What is the range of $f(x)$?
f) Sketch $f(x)$.

II Given $f(x)$, find $f'(x)$: (5 pts ea)

a) $f(x) = e^{\cos x}$

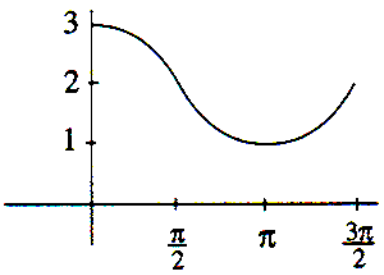
b) $f(x) = x^{e^x}$

c) $f(x) = \ln e^{5x}$

d) $f(x) = \ln\left(\dfrac{\sin x}{\cos x}\right)$

III Find the x-intercept of the line tangent to $f(x) = e^x$ which has slope 2. (15 pts)

IV Given $y = \cos x + 2$ graphed on the right. (15 pts)
Find the area trapped below y, above the x-axis and between $x = \dfrac{\pi}{3}$ and $x = \dfrac{4\pi}{3}$.



V Jasper says, "Anything to the zero power is 1." (15 pts)

Jed says, "Not so!". Jed writes $y = x^{\frac{1}{\ln x}}$ on the chalkboard and says, "As x gets large, $\ln x$ also gets bigger so $\dfrac{1}{\ln x}$ is certainly positive and is getting closer to zero." He continued, "So we can say $\lim\limits_{x\to\infty} x^{\frac{1}{\ln x}}$ is an example of ∞^0 which is *not* 1."
Explain what Jed meant and show that he was correct.

Extra Credit 5 pts

How old were you at thirty-four minutes fifty-six seconds past noon on July 8, 1990? What was special about that particular time?

I

$$f(x) = \begin{cases} 0 & \text{if } x = 0 \\ x^2 \ln x & \text{if } 0 < x \le 2 \end{cases}$$

(u v)

a) $f'(x) = 2x\ln x + \frac{x^2}{x}$

$= 2x\ln x + x \overset{\text{set}}{=} 0$

(u v)

$(2\ln x + 1)x = 0$

$\ln x = -\frac{1}{2} \quad x = 0$

$x = e^{-\frac{1}{2}}$

critical value $e^{-\frac{1}{2}}$

b) f': $-$ on $(0, \frac{1}{\sqrt{e}})$, $+$ on $(\frac{1}{\sqrt{e}}, 2)$

f decreases for $0 < x < \frac{1}{\sqrt{e}}$

c) $f''(x) = x \cdot \frac{2}{x} + 2\ln x + 1$

$= 3 + 2\ln x \overset{\text{set}}{=} 0$

$\ln x = -\frac{3}{2}$

$x = e^{-\frac{3}{2}}$

f'': $-$ on $(0, \frac{1}{\sqrt{e^3}})$, $+$ on $(\frac{1}{\sqrt{e^3}}, 2)$

f concave down for $0 < x < \frac{1}{\sqrt{e^3}}$

d) $(0,0)$ MAX ENDPT discont.

$(\frac{1}{\sqrt{e}}, \frac{-1}{2e})$ min min f' test.

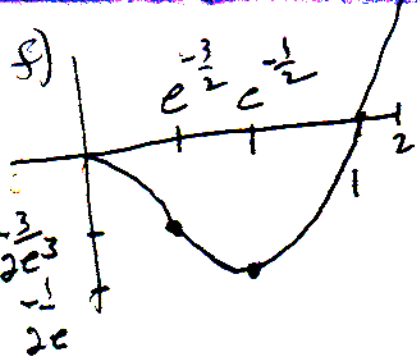
$(2, 4\ln 2)$ max ENDPT inc to it.

e) $f(e^{-\frac{3}{2}}) = e^{-3}\ln(e^{-\frac{3}{2}})$

$= \frac{-3}{2e^3}$

PT at $(e^{-\frac{3}{2}}, \frac{-3}{2e^3})$

f)



II

a) $f(x) = \cos e^{3x}$

$f'(x) = -\sin e^{3x}(3e^{3x})$

$= -3e^{3x}\sin e^{3x}$

b) $f(x) = \ln e^{3x^2}$

$= 3x^2$

$f'(x) = 6x$

c) $f(x) = x^{e^x}$

$\ln y = e^x \ln x$ (u v) — no product rule -5

$\frac{1}{y}\frac{dy}{dx} = e^x \ln x + \frac{e^x}{x}$ — error on product rule -3

$\frac{dy}{dx} = x^{e^x}\left(e^x \ln x + \frac{e^x}{x}\right)$

$= \frac{e^x x^{e^x}(x\ln x + 1)}{x}$

b) $f(x) = \frac{1}{\ln x} = (\ln x)^{-1}$

$f'(x) = -(\ln x)^{-2}\frac{1}{x}$

$= \frac{-1}{x(\ln x)^2}$

1
1 1
1 2 1
1 3 3 1

III a) $\int \frac{(3-2x)^3}{x}dx = \int \frac{3^3 - 54x + 12x^2 - 8x^3}{x}dx$

don't do c) → −7

$= \int \frac{27}{x} - 54 + 12x - 8x^2\,dx =$

$= 27\ln|x| - 54x + 6x^2 - \frac{8}{3}x^3 + C$

b) ↓

VII 6) $\int e^{-3x}\,dx$

$-\frac{1}{3}e^{-3x}+C$

VIII a) $\ln 40$
$\ln 8.5$
$3\ln 2+\ln 5$
$3+c$

b) $\log_3 8$
$\frac{3\ln 2}{\ln 3}$
$\frac{3a}{b}$

c) $\ln 2.4 =$
$\ln \frac{12}{5}$
$2\ln 2+\ln 3-\ln 5$
$2a+b-c$

d) $\log_5 2 \cdot \log_2 25$
$\frac{\ln 2}{\ln 5}\cdot\frac{2\ln 5}{\ln 2}$
2

IX

$f(x)=e^x$

$m=e^2 \quad f'=e^x \overset{SA}{=} e^2$

$\therefore x=2$

$(2, e^2), m=e^2$

$y-e^2 = e^2(x-2)$

$= e^2x-2e^2+e^2$

$y=e^2x-e^2 \overset{SA}{=} 0$

$x-1=0$

$x=1$

x intercept is $(1,0)$

extra.

$y=\frac{x^2}{x^2+1}$ u

$y'=\frac{(x^2+1)2x-x^2\cdot 2x}{(x^2+1)^2}$

$=\frac{2x^3+2x-2x^3}{(x^2+1)^2}$

$=\frac{2x}{(x^2+1)^2}$ u

$y''=\frac{2(x^2+1)^2-2x\cdot 2(x^2+1)2x}{(x^2+1)^4}$

$=\frac{2(x^2+1)(x^2+1-4x^2)}{(x^2+1)^4}$

$=\frac{2(x^2+1)(1-3x^2)}{(x^2+1)^4} \overset{SA}{=} 0$

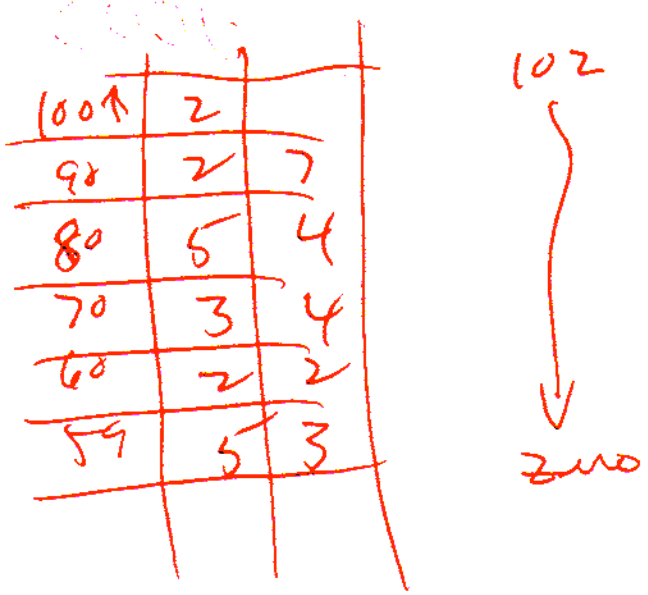
$1-3x^2 \overset{SA}{=} 0$

$x^2=\frac{1}{3}$

$x=\pm\frac{1}{\sqrt{3}} \quad y=\frac{\frac{1}{3}}{\frac{4}{3}}=\frac{1}{4}$

$\frac{1}{p^2}+\frac{1}{q^2}=$

$3+16=19$



$\frac{1305}{17}$ $\frac{1538}{19}$ 80.1

$\frac{1266}{16}$ 80.9

79.1

A good score for the end of the year. Those who did not fail but those who would repeat the benefits.

Honors Analysis

Name ANL Ch 5&7

Per _____ Date 2003

Copy original problem.

Convince *me* that **you** understand the concept!

No Calculators!

Chapters 5, 6, & 7 Exam

I Given: $f(x)=\begin{cases}0 & \text{if } x=0\\ x^2\ln x & \text{if } 0<x\le 2\end{cases}$ (30 pts tot)

a) Find the cirtical values (Zeros of $f'(x)$

b) Find the interval(s) where $f(x)$ is decreasing.

c) Find the interval(s) where $f(x)$ is concave down.

d) Find, identify and verify the coordinates of *all* extrema.

e) Find the coordinates of any point(s) of inflection.

f) Draw $f(x)$. Be sure to specifically indicate any significant points.

II Given $f(x)$, find $f'(x)$: (5 pts ea)

a) $f(x)=\cos(e^{3x})$ b) $f(x)=\ln(e^{3x^2})$ c) $f(x)=x^{e^x}$ d) $f(x)=\dfrac{1}{\ln x}$

III Find the integral: (10 pts ea)

a) $\int\dfrac{(3-2x)^3}{x}\,dx$ b) $\int e^{-3x}\,dx$

IV Simplify using the "Mumaugh" tables: (5 ps ea)

a) $\ln 40$ b) $\log_3 8$ c) $\ln(2.4)$ d) $(\log_5 2)(\log_2 25)$

V Find the x-intercept of the line tangent to $f(x)=e^x$ which has slope e^2. (10 pts)

Extra Credit ---------- 5 pts ----------

The curve $y=\dfrac{x^2}{x^2+1}$ has two inflection points. They are $(\pm p,q)$. Find the value of : $\dfrac{1}{p^2}+\dfrac{1}{q^2}$.

I

$a = -k$

$\frac{dv}{dt} = -k$

$\int dv = \int -k\, dt$

$v = -kt + C \quad T=0\ V=66$

$\therefore C = 66$

$v = \frac{ds}{dt} = -kt + 66$

$\int ds = \int -kt + 66\ dt$

$s = -\frac{kt^2}{2} + 66t + C \quad T=0\ S=0$

$\therefore C = 0$

$s = -\frac{kt^2}{2} + 66t$

$v = 0$ car is stopped.

$\therefore 0 = -kt + 66$

$t = \frac{66}{k}$

when $s = 60$ car is stopped.

$60 = -\frac{k}{2}\left(\frac{66^2}{k^2}\right) + 66\left(\frac{66}{k}\right)$

$60k = -\frac{66^2}{2} + 66^2$

$60k = \frac{66^2}{2}$

$k = \frac{33 \cdot 66 \cdot 66}{2 \cdot 60 \cdot 10} = \frac{363}{10}$

II $y = \ln(4 + x^2) \quad x \in [-4, 4]$

a) $y' = \frac{2x}{4+x^2} \overset{SV}{=} 0 \quad x = 0$

y'	$-$	0	$+$
		$\frac{1}{3}$	

$y|_{x=0} = \ln 4 \qquad y|_{x=\pm 4} = \ln 20$

$(-4, \ln 20)$ abs max dec from it

$(0, \ln 4)$ abs min f' ~~tut~~

$(4, \ln 20)$ abs max inc to it.

b) $y' = \frac{2x}{4+x^2} \cdot \frac{4}{r}$

$y'' = \frac{(4+x^2)2 - 2x(2x)}{(4+2x)^2} \overset{SV}{=}$

$8 + 2x^2 - 4x^2 = 0$

$2x^2 = 8$

$x = \pm 2$

PI

$(-2, \ln 8), (2, \ln 8)$

y''	$-$	0	$+$	0	$-$
		-2	0	2	

III

$$\frac{dy}{dx} = \frac{2y}{x}$$

$$\int \frac{dy}{y} = \int \frac{2}{x}\,dx$$

$$\ln y = 2\ln x + c$$

$$e^{\ln y} = e^{\ln(x^2+c)}$$

$$y = x^2 + c$$

$$1 = 1 + c$$

$$\therefore c = 0$$

$$y = x^2$$

IV

$$v = 3t^{\frac{1}{2}} + 5t^{\frac{3}{2}}$$

$$\int ds = \int 3t^{\frac{1}{2}} + 5t^{\frac{3}{2}}\,dt$$

$$s = 2t^{3/2} + 2t^{\frac{5}{2}} + c$$

$t = 0 \quad s = 0 \quad \therefore c = 0$

$t = 4$

$$s = 2 \cdot 8 + 2 \cdot 32$$

$$16 + 64 = 80$$

81
243

$t = 9$

$$s = 2 \cdot 27 + 486$$

$$54 \qquad \frac{54}{540}$$

$$-80$$

$$460 \text{ feet.}$$

V

A) $y = 10^{x^2-1}$

$$\ln y = (x^2-1)\ln 10$$

$$\frac{1}{y}\frac{dy}{dx} = 2x\ln 10$$

$$\frac{dy}{dx} = 2x\ln 10\,(10^{x^2-1})$$

B) $y = e^{x^e}$

$$y' = e^{x^e}(ex^{e-1})$$

C) $y = \ln\left(\frac{1}{e}\right)$

$$y' = 0$$

D) $y = \log_2(e^x)$

$$y = x\log_2 e$$

$$y' = \log_2 e \quad \text{or} \quad \frac{1}{\ln 2}$$

$$y = \frac{\ln e^x}{\ln 2}$$

$$\frac{x}{\ln 2}$$

$$y' = \frac{1}{\ln 2}$$

VI

A) $\int \sin x + e^x\,dx =$

$$-\cos x + e^x + c$$

B) $\int (3x^{-2} + x)^2\,dx =$

$$\int 9x^{-4} + 6x^{-1} + x^2\,dx =$$

$$\frac{9x^{-3}}{-3} + 6\ln|x| + \frac{x^3}{3}$$

$$-3x^{-3} + 6\ln|x| + \frac{x^3}{3}$$

1987

ANALYSIS CHAPTERS 6, 7 & 8 Name C 678 make up

Convince ME that you know what you are doing.

Per _____

MAKE UP

I A car is traveling on a straight road at 45 miles per hour (66 feet per second).
The car slows down at the constant rate of k feet per second and comes to a stop after traveling 60 feet.
Find k. (hint: Notice that at t=0, V=66 AND S=0) (20 pts)

II Given $Y = \text{Ln}(4 + X^2)$ and $X \in [-4, 4]$ (20 pts tot)

A) Determine the coordinates of all extrema.

B) Determine the coordinates of any Points of Inflection.

III Find the equation of the curve whose slope at any point (X,Y) equals $\frac{2Y}{X}$ and which passes through the point (1,1). (15 pts)

IV The **Velocity** of a particle moving on a line at time **t** is : $V = 3t^{\frac{1}{2}} + 5t^{\frac{3}{2}}$ feet per second.
The particle started at zero when **t** = 0.
How far did the particle travel from **t** = 4 to **t** = 9 ? (15 pts)

V Given Y, find Y' : (5 pts ea)

A) $Y = 10^{(X^2 - 1)}$ B) $Y = e^{X^e}$ C) $Y = \text{Ln}(\frac{1}{e})$

D) $Y = \text{Log}_2(e^X)$

VI Find the integral, as indicated (5 pts ea)

A) $\int \text{Sin } X + e^X \, dX$ B) $\int (3X^{-2} + X)^2 \, dX$

I

A) $y = 3x^{5e}$

$y' = 15e x^{5e-1}$

B) $y = 3e^{5x}$

$y' = 15e^{5x}$

C) $y = e^{3^x}$

$\ln y = \ln e + x \ln 3$

$\frac{1}{y}\frac{dy}{dx} = \ln 3$

$\frac{dy}{dx} = e 3^x \ln 3$

D) $y = 3^{x^e}$

$\ln y = x^e \ln 3$

$\frac{1}{y}\frac{dy}{dx} = e x^{e-1} \ln 3$

$\frac{dy}{dx} = 3^{x^e} e x^{e-1} \ln 3$

E) $y = 3^{e^x}$

$\ln y = e^x \ln 3$

$\frac{1}{y}\frac{dy}{dx} = e^x \ln 3$

$\frac{dy}{dx} = 3^{e^x} e^x \ln 3$

F) $y = \ln 3x$

$y' = \frac{1}{x}$ derivative of ln is 1/x; derivative of 3x is 3

G) $y = (e^{\frac{1}{x}})^2 = e^{\frac{2}{x}}$

$y' = e^{\frac{2}{x}}\left(\frac{-2}{x^2}\right)$

wrong problem

correctly applied chain rule

also mean -4

H) $y = \log_3 x^e = \frac{e \ln x}{\ln 3}$

$y' = \frac{e}{x \ln 3}$

II

A) $\int_1^e \frac{(3x-2)^3}{x^2}\,dx$

$9x^2 - 12x + 4$

$3x - 2$

$-18x^2 + 24x - 8$

$27x^3 - 36x^2 + 12x$

$27x^3 - 54x^2 + 36x - 8$

$\int_1^e 27x - 54 + \frac{36}{x} - \frac{8}{x^2}\,dx$

not evaluated $= -4$

144

27

137

$\frac{27x^2}{2} - 54x + 36\ln x + \frac{8}{x}\Big|_1^e$

$\frac{27e^2}{2} - 54e + 36 + \frac{8}{e} - \left[\frac{27}{2} - 54 + 0 + 8\right] = \frac{27e^2}{2} - 54e + \frac{8}{e} + \frac{137}{2}$

B) $\int_0^{\frac{\pi}{4}} \sin 2x\,dx = -\frac{1}{2}\cos 2x\Big|_0^{\frac{\pi}{4}} = -\frac{1}{2}\left[0 - 1\right] = \frac{1}{2}$

III $e^y = x$

$y = \ln x$

$y' = \frac{1}{x}$

$y'' = -\frac{1}{x^2}$

IV $f(x) = e^x$

$f'(x) = e^x \overset{\text{set}}{=} e^2 \quad \therefore x = 2 \quad (2, e^2)$

$y - e^2 = e^2(x-2)$

$y - e^2 = e^2 x - 2e^2$ set $= 0$ 3

$y = e^2 x - e^2$ $x = 1$ 3pts

V

$P = .4Nx^2$ $\quad N = 4$ $\quad x = 3\frac{1}{4} = \frac{13}{4}$ $\quad \Delta x = \frac{15}{1000}$

$df = f'(x)\,\Delta x$

$= .8Nx\,\Delta x$

$= \frac{8}{10} \cdot 4 \cdot \frac{13}{4} \cdot \frac{15}{1000}$

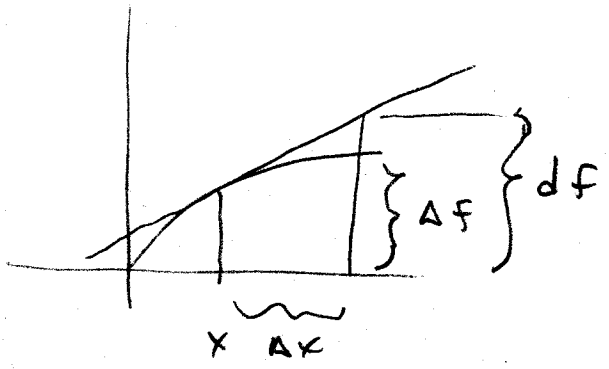
$= \frac{156}{1000}$

$\frac{144}{12}$

Extra credit

df is the approximation

Δf is the actual change in value from x to $x+\Delta x$



df is greater than Δf if f is concave down in the vicinity of $x+\Delta x$.

89 ◎

make the 2 changes indicated

	2	5	T	
100	1	0	1	101
90	4	2	3	
80	4	5	9	
70	6	3	9	
60	8	2	10	
59	3	4	7	50
	72.1	75.5	73.4	

ANALYSIS Ch 5-8

Honors ANALYSIS NAME _____

Convince ME that *YOU* know the concept ! PER _____ DATE 6/9/88

Chapters 5, 6, 7, 8 EXAM

I Given $F(x)$, find $F'(x)$ (5 pts each)

A) $3x^{5e}$ B) $3e^{5x}$ C) $e3^{x}$ D) $3^{x^{e}}$ E) $3^{e^{x}}$

F) $\text{Ln}3x$ G) $\left(\frac{1}{e^{x}}\right)^{2}$ H) $\text{Log}_{3}x^{e}$

II Find the integral, and evaluate as indicated : (10 pts each)

A) $\int_{1}^{e} \frac{(3x-2)^{3}}{x^{2}}\,dx$ B) $\int_{0}^{\frac{\pi}{4}} \text{Sin}2x\,dx$

III Given : $e^{y} = x$. Find the first and second derivatives of y with respect to x . (15 pts total)

IV Find the x-intercept of the line tangent to $F(x) = e^{x}$ which has slope e^{2} . (15 pts)

V The expression for the horsepower of an engine is : $P = 0.4nx^{2}$, where n is the number of cylinders and x is the bore (diameter) of the cylinders. Determine the approximate increase in power (the differential) when a (4 cylinder) Volkswagon motor has the cylinders rebored from 3.250 to 3.265 inches. (10 pts)

EXTRA CREDIT 5 pts

Explain the difference between df and Δf . Under what conditions will df be greater than Δf ? (You must be very explicit for full credit.)

I

$f(x) = \ln x \quad (x, \ln x)$

$\frac{1}{x} = e^2 \therefore x = e^{-2} \therefore (e^{-2}, -2) \quad m = e^2$

$y + 2 = e^2(x - e^{-2})$

$y + 2 = e^2 x - 1$

A) $y = e^2 x - 3$ 10

B) $G(x) = e^x \quad (x, e^x)$

$(2, e^2) \quad m = e^2$ $\quad y - e^2 = e^2(x-2)$

$y = e^2 x - e^2$ 10

Problems in A/B because C) work but no Sol = -3

C) $e^2 x - 3 \overset{\text{Sub}}{=} e^2 x - e^3$

$3 = e^3$ Reject. The lines are ∥. No intersection 10

II

A) $f(x) = \ln\left(\frac{1}{x^2}\right)$

$= -2\ln x$

Not reduced $f' = \frac{-2}{x}$ ✓

B) $f(x) = \frac{e^x}{x}$ u v

$f'(x) = \frac{xe^x - e^x}{x^2}$

$= \frac{e^x}{x^2}(x-1)$ ✓

C) $f(x) = e^{-x^3}$

$f'(x) = -3x^2 e^{-x^3}$ ✓

D) $f(x) = x^2 e^x$ u v

$f' = 2xe^x + x^2 e^x$ ✓

E) $f(x) = x^e$

$f' = e x^{e-1}$ ✓

F) $f(x) = \frac{1}{\ln x}$

$= (\ln x)^{-1}$

$f' = -(\ln x)^{-2} \cdot \frac{1}{x}$

$= \frac{-1}{x \ln^2 x}$ ✓

III

A) $\int \frac{x^2 + x + 1}{x} dx = \int x + 1 + \frac{1}{x} dx = \frac{x^2}{2} + x + \ln|x| + C$ ✓

B) $\int \frac{(x^4+1)(x^2+1)}{x^3} dx = \int \frac{x^6 + x^4 + x^2 + 1}{x^3} dx$

$\int x^3 + x + \frac{1}{x} + \frac{1}{x^3} dx = \frac{x^4}{4} + \frac{x^2}{2} + \ln|x| - \frac{1}{2x^2} + C$ ✓

C) $\int e^{4x} dx = \frac{e^{4x}}{4} + C$ ✓

D) $\int (2x^3 + 1)^2 dx = \int 4x^6 + 4x^3 + 1 \, dx =$

$\frac{4x^7}{7} + x^4 + x + C$ ✓

10 $z = ce^{kt}$

$\frac{dz}{dt} = cke^{kt} = kce^{kt}$ — 5

$= kz$. — 10

11 $f(x) = \tan x - \ln x^{\pi}$

$f(x) = \frac{\sin x}{\cos x} - \pi \ln x$

$f'(x) = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} - \frac{\pi}{x}$

$= \frac{1}{\cos^2 x} - \frac{\pi}{x}\Big|_{x=\pi} = \frac{1}{1} - \frac{\pi}{\pi} = 0$ — 10

EXTRA.

$$\frac{5x-11}{2x^2+x-6} = \frac{A}{x+2} + \frac{B}{2x-3}$$

$5x-11 = A(2x-3) + B(x+2)$

$x=-2$ $\quad -21 = -7A \quad \therefore A = 3$

$x=0$ $\quad -11 = -3A + 2B$

$-11 = -3 \cdot 3 + 2B$

$-2 = 2B \quad \therefore B = -1$

$$\int \frac{3\,dx}{x+2} - \int \frac{dx}{2x-3}$$

$3\ln|x+2| - \frac{1}{2}\ln|2x-3| + C$

1985

	5	6	T
100↑	0	1	1
90	3	5	8
80	6	9	15
70	11	2	13
60	7	4	11
59	4	1	5
	72.4	82.2	76.6

100 → 35

make change as shown on reverse side.

1987

	2	5	T
100	0	1	1
90	2	3	5
80	2	1	3
70	3	3	6
60	2	5	7
59	15	8	23
	60.5	69.9	63.5

101 → 37

one additional change.

this class never could figure out the equation of a straight line — & probably never will —

Calculus 1987-88 will be a drag! IT was!

Probably even less able that the 88-89 group! will see.

ANALYSIS CHAPTERS 6, 7 & 8 Name Analysis Ch 6,7,8

Convince ME that you know what you are doing. Per 6/7/85 Key

used 1987

I Given: L_1 is tangent to $F(x) = \ln x$ and has slope e^2 and L_2 is tangent to $G(x) = e^x$ at $x = 2$, (10 pts ea)

A) Determine the equation of L_1.

B) Determine the equation of L_2.

C) Determine the coordinates of intersection, if any, of L_1 and L_2. 5/29/87 used 90

II Given $F(x)$, find $F'(x)$: (5 pts ea)

$\ln\left(\frac{1}{x^2}\right)$

A) $F(x) = \ln\frac{1}{x^2}$ B) $F(x) = \frac{e^x}{x}$ C) $F(x) = e^{-x^3}$

D) $F(x) = x^2e^x$ E) $F(x) = x^e$ F) $F(x) = \frac{1}{\ln x}$

III Find the integral, as indicated (5 pts ea)

A) $\int \frac{x^2 + x + 1}{x}\,dx$ B) $\int \frac{(x^4 + 1)(x^2 + 1)}{x^3}\,dx$ C) $\int e^{4x}\,dx$ D) $\int (2x^3 + 1)^2\,dx$

IV Given $Z = C\,e^{kt}$, where C, k are constants.

Find $\frac{dz}{dt}$. Your final answer MUST NOT contain the constant e. (10 pts)

V Given $F(x) = \tan x - \ln x^{\pi}$, Find $F'(\pi)$ (10 pts)

changed to + 5/30/87

EXTRA CREDIT ——— 5 PTS ———

Given: $\frac{5x - 11}{2x^2 + x - 6} = \frac{A}{x + 2} + \frac{B}{2x - 3}$

Determine A and B, then find the integral of the right side of the equation.

I

A) $f(x) = \ln x$

$f'(x) = \frac{1}{x} \overset{\text{Sub}}{=} e^2$

$x = e^{-2}$

$(e^{-2}, -2)$

$y + 2 = e^2(x - e^{-2})$

$y + 2 = e^2 x - 1$

$y = e^2 x - 3$

B) $G(x) = e^x$

$G'(x) = e^x$

$(2, e^2)$

$m = e^2$

$y - e^2 = e^2(x - 2)$

$y - e^2 = e^2 x - 2e^2$

$y = e^2 x - e^2$

C) L_1 and L_2 Do not intersect since they are $\parallel$ lines with different y intercepts

II

A) $f(x) = \ln \frac{1}{x^2}$

$= -2 \ln x$

$f'(x) = -\frac{2}{x}$

B) $f(x) = \frac{e^x}{x}$ u v

$f'(x) = \frac{x e^x - e^x}{x^2}$

C) $f(x) = e^{-x^3}$

$f'(x) = -3x^2 e^{-x^3}$

D) $f(x) = x^2 e^x$ u v

$f'(x) = 2x e^x + x^2 e^x$

E) $f(x) = x^e$

$f'(x) = e x^{e-1}$

F) $f(x) = \frac{1}{\ln x} = (\ln x)^{-1}$

$f'(x) = -(\ln x)^{-2} \frac{1}{x}$

$= \frac{-1}{x(\ln x)^2}$

III

A) $\int_1^e \frac{(x^4+1)(x^2+1)}{x^3}\,dx$

$\int_1^e \frac{x^6 + x^4 + x^2 + 1}{x^3}\,dx$

$\int_1^e x^3 + x + \frac{1}{x} + \frac{1}{x^3}\,dx$

$\frac{x^4}{4} + \frac{x^2}{2} + \ln x - \frac{x^{-2}}{2}\Big|_1^e$

$\frac{e^4}{4} + \frac{e^2}{2} + 1 - \frac{1}{2e^2} - \left[\frac{1}{4} + \frac{1}{2} + 0 - \frac{1}{2}\right]$

$\frac{e^4}{4} + \frac{e^2}{2} + \frac{3}{4} - \frac{1}{2e^2}$

B) $\int_0^1 e^{4x}\,dx$

$\frac{1}{4} e^{4x}\Big|_0^1$

$\frac{1}{4} e^4 - \frac{1}{4}$

C) $\int_{-1}^1 (2x^3 + 1)^2\,dx$

$\int_{-1}^1 4x^6 + 4x^3 + 1\,dx$

$\frac{4}{7}x^7 + x^4 + x\Big|_{-1}^1$

$\frac{4}{7} + 1 + 1 - \left[-\frac{4}{7} + 1 - 1\right]$

$\frac{4}{7} + 1 + 1 + \frac{4}{7} + 1 - 1$

$\frac{8}{7} + 2$

$\frac{22}{7}$

IV $f(x) = \frac{\ln x}{x}$ u $\quad$ $\frac{1-\ln x}{x^2}$ v

$f'(x) = \frac{x\cdot\frac{1}{x} - \ln x}{x^2} \overset{set}{=} 0$

f': + 0 − (0, e)

$\ln x = 1$

$x = e$

$(e, \frac{1}{e})$ max

$f''(x) = \frac{x^2\left(-\frac{1}{x}\right) - (1-\ln x)2x}{x^4}$

dN up

f'': − 0 + (0, $e^{\frac{3}{2}}$)

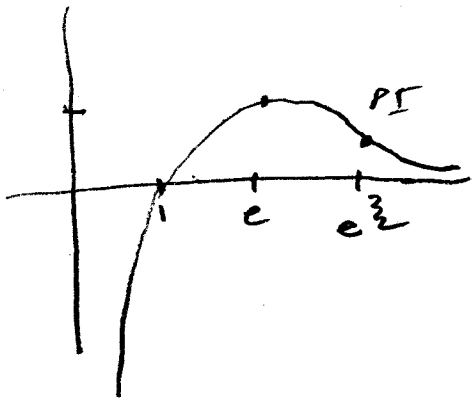
$= \frac{-x - 2x + 2x\ln x}{x^4}$

$= \frac{-3 + 2\ln x}{x^3} \overset{set}{=} 0$

$\ln x = \frac{3}{2}$

$x = e^{\frac{3}{2}}$

$\left(e^{\frac{3}{2}}, \frac{3}{2e^{\frac{3}{2}}}\right)$ PI



I $f(x) = \tan x + \ln x^{\pi}$

$f'(x) = \sec^2 x + \frac{\pi}{x}\Big|_{x=\pi}$

$1 + \frac{\pi}{\pi} = 2$

EXTRA.

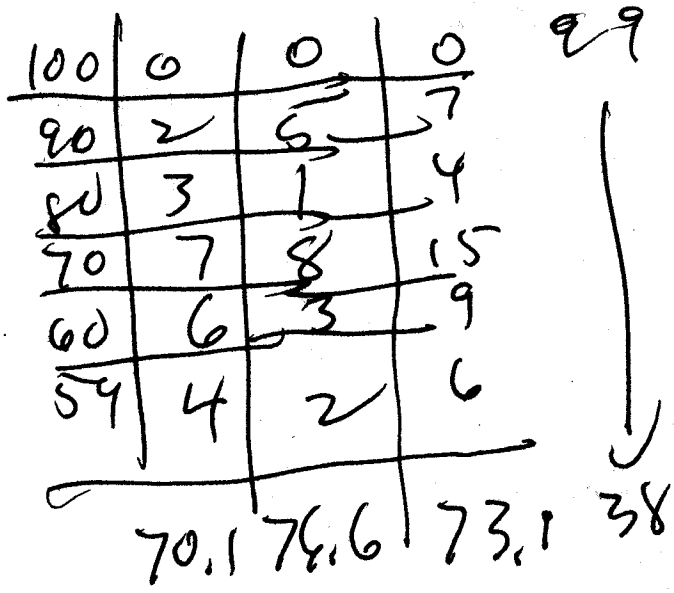
$z = ce^{kt}$

$\frac{dz}{dt} = cke^{kt}$

$= k(ce^{kt})$

$= kz$

1990



Excellent exam.
Students "shut down" about a week or so too early.

No changes

HONORS ANALYSIS
Convince ME that YOU know the concept !
Copy original problem. SHOW ALL WORK !

NAME ANL C5678 90

PER _____ DATE _____

LAST REGULAR EXAM

I Given: L_1 and L_2 are straignt lines. (25 pts tot)

L_1 is tangent to $F(x) = \text{Ln}\, x$ and has slope e^2 and

L_2 is tangent to $G(x) = e^x$ at $x = 2$.

A) Determine the equation of L_1.

B) Determine the equation of L_2.

C) Determine the coordinates of intersection, if any, of L_1 and L_2.

II Given $F(x)$, find $F'(x)$: (5 pts ea)

A) $F(x) = \text{Ln}\, \frac{1}{x^2}$ B) $F(x) = \frac{e^x}{x}$ C) $F(x) = e^{-x^3}$

D) $F(x) = x^2 e^x$ E) $F(x) = x^e$ F) $F(x) = \frac{1}{\text{Ln}\, x}$

III Find the integral as indicated: (5 pts ea)

A) $\int_1^e \frac{\left(x^4+1\right)\left(x^2+1\right)}{x^3}\, dx$ B) $\int_0^1 e^{4x}\, dx$ C) $\int_{-1}^1 \left(2x^3+1\right)^2\, dx$.

IV Given $F(x) = \frac{\text{Ln}\, x}{x}$. Draw the graph of $F'(x)$. Locate and label *all* significant points. Justify, of course. (20 pts)

V Given $F(x) = \text{Tan}\, x + \text{Ln}\, x^{\pi}$. Find $F'(\pi)$. (10 pts)

Extra Credit ═══ 5 pts ═══

Given: $Z = Ce^{kt}$. Where C and k are constants

Find $\frac{dZ}{dt}$. Your final answer MUST *NOT* CONTAIN the constant e.

I

a) $y = x^{e^x}$

$\ln y = e^x \ln x$

$\frac{1}{y}\frac{dy}{dx} = e^x \ln x + \frac{e^x}{x}$

$\frac{dy}{dx} = x^{e^x}\left[\frac{xe^x \ln x + e^x}{x}\right]$

b) $y = x^{x^e}$

$\ln y = x^e \ln x$

$\frac{1}{y}\frac{dy}{dx} = ex^{e-1}\ln x + \frac{x^e}{x}$

$\frac{dy}{dx} = x^{x^e}\left[\frac{ex^e \ln x + x^e}{x}\right]$

c) $y = e^{x^e}$

$y' = e^{x^e} \cdot ex^{e-1}$

$y' = e^{x^e+1}x^{e-1}$

d) $y = e^{e^e}$

$y' = 0$

e) $y = e^{e^x}$

$y' = e^{e^x}e^x$

f) $y = \log_{10} x^e$

$= \frac{e \ln x}{\ln 10}$

$y' = \frac{e}{x \ln 10}$

g) $y = \log_{10} e^x$

$y = x \log_{10} e$

$y' = \log_{10} e$

II

$f(x) = xe^x \quad [-2, e]$

$f' = e^x + xe^x \overset{\text{set}}{=} 0$

$= e^x(1+x)$

$e^x \neq 0 \quad x = -1$

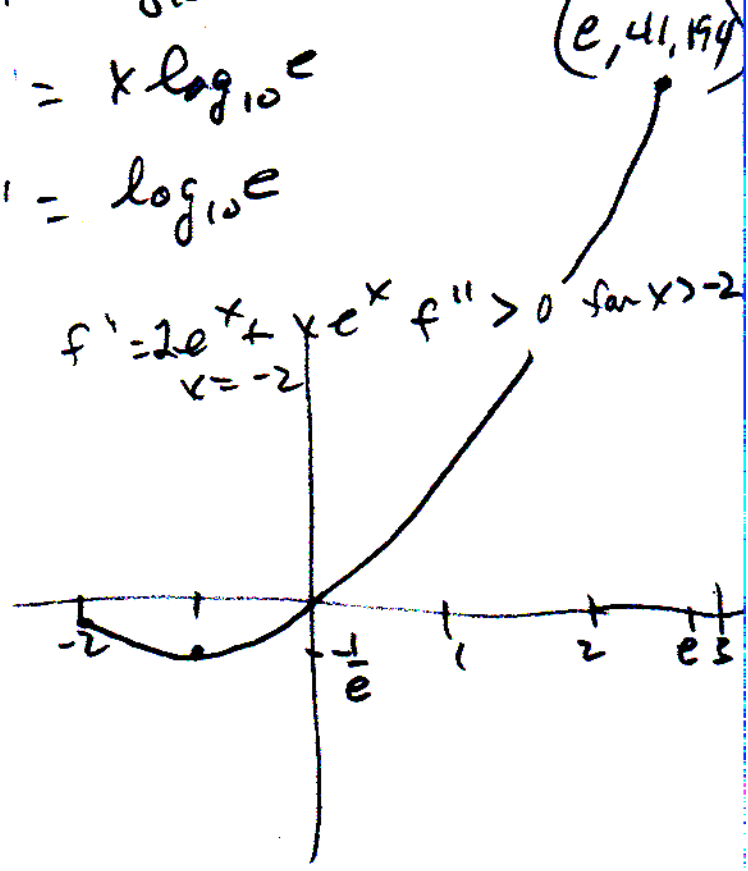
	-2		-1		e
f'		$-0+$			
f		↓		↑	
f''			$+$		
			↑		

$(-1, \frac{-1}{e})$ min f'tn.

$(-2, \frac{-2}{e^2})$ max endpt dec fn'l.

(e, e^{e+1}) max endpt INC fn'l.

$(2.718, 41.194)$



15 a) $\int (2-3x^5)^3\,dx$

1
1 1
1 2 1
1 3 3 1

$\int 1\cdot 2^3 - 3\cdot 2^2(3x^5) + 3\cdot 2(3x^5)^2 + (3x^5)^3\,dx$

$\int 8 - 36x^5 + 54x^{10} - 27x^{15}\,dx$

$8x - 6x^6 + \frac{54x^{11}}{11} - \frac{27}{16}x^{16} + C$

b) $\int \frac{2x^3+x^2+3x+2}{x}\,dx = \int 2x^2 + x + 3 + \frac{2}{x}\,dx$

$\frac{2}{3}x^3 + \frac{1}{2}x^2 + 3x + 2\ln|x| + c$

c) $\int \frac{1}{\cos^2 x}\,dx = \int \sec^2 x\,dx$

$= \tan x + C.$

16 $y = \ln|x|$

If $x > 0$ $\quad y = \ln x$

$y' = \frac{1}{x}$

If $x < 0$ $\quad y = \ln(-x)$

$y' = \frac{1}{-x}(-1)$

$= \frac{1}{x}$

If $y \neq 0$ then $y' = \frac{1}{x}$.

Extra.

12:34.56.7.8.90 were the time and date designations

limits on # should be $[-3, e]$ so PI at -2 shows up.

95

100	1	0	1
90	3	3	6
80	4	1	5
70	5	2	7
60	3	3	6
59	1	3	4

100

$\frac{1291}{16}$ $\frac{768}{10}$ $\frac{2057}{26}$ 20

80.7 ~~696~~ 76.8 79.7

Honors Analysis

Copy original problem.

Convince *me* that **you** understand the concept!

Name AnL C567

Per _____ Date June 95

Last Regular Exam

I Given y, find $\frac{dy}{dx}$. (60 pts tot)

a) $y = x^{e^x}$ b) $y = x^{x^e}$ c) $y = e^{x^e}$ d) $y = e^{e^e}$

e) $y = e^{e^x}$ f) $y = \log_{10} x^e$ g) $y = \log_{10} e^x$

II Determine the coordinates of the extrema. Sketch. Justify. Remember that a picture is not justification.

$f(x) = xe^x; \quad D_f \quad x \in [-2, e]$ (15 pts)

III Compute the antiderivative. Remember to include the constant of integration. (5 pts ea)

a) $\int (2 - 3x^5)^3 \, dx$ b) $\int \frac{2x^3 + x^2 + 3x + 2}{x} \, dx$

c) $\int \frac{1}{\cos^2 x} \, dx$

IV Show that the derivative of $y = \ln|x|$ for $x \neq 0$ is $\frac{1}{x}$. (Note: A proof is not required.) (10 pts)

Extra Credit ---------- 5 pts ----------

What was special about the time thirty-four minutes, fifty-six seconds past noon on July 8, 1990?

I $f(x) = \frac{\ln x}{x}$

a) D_f $x > 0$

b) $f(x) = \frac{\ln x}{x}$ u / v

$f'(x) = \frac{x\cdot\frac{1}{x} - \ln x}{x^2}$

$f'(x) = \frac{1 - \ln x}{x^2} \overset{\text{set}}{=} 0$

$x = e$

$(e, \frac{1}{e})$

f'	+ 0 −
	0 e

c) only max $(e, \frac{1}{e})$

f' test

d) $f'(x) = \frac{1-\ln x}{x^2}$ u / v

$f''(x) = \frac{x^2(-\frac{1}{x}) - 2x(1-\ln x)}{x^4}$

$= \frac{-x - 2x + 2x\ln x}{x^4}$

$= \frac{-3 + 2\ln x}{x^3} \overset{\text{set}}{=} 0$

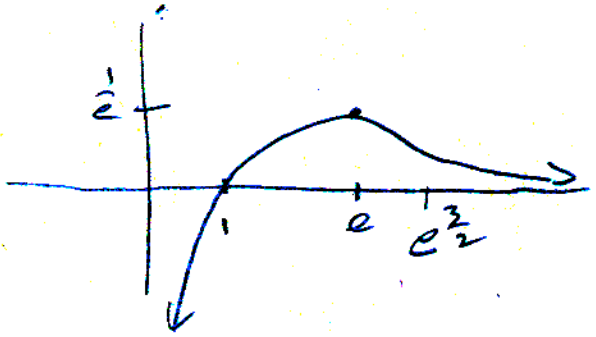
$2\ln x = 3$

$x = e^{\frac{3}{2}}$

$(e^{\frac{3}{2}}, \frac{3}{2e^{\frac{3}{2}}})$

f''	− 0 +
	0 $e^{\frac{3}{2}}$

e) f increases $0 < x < e$



II

a) $f(x) = \sin e^{-2x}$ 2 pts

$f'(x) = -2e^{-2x}\cos e^{-2x}$

b) $f(x) = \ln(\tan x)$

$= \ln(\sin x) - \ln(\cos x)$

OR

$\frac{1}{\tan x}\cdot\sec^2 x$ $f' = \frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}$

$\cot x \sec^2 x = \frac{\cos^2 x + \sin^2 x}{\sin x \cos x}$

$= \csc x \sec x$

c) $f(x) = e^{x^e}$

$f'(x) = e^{x^e}(e x^{e-1})$

d) $f(x) = \frac{1}{\ln x} = (\ln x)^{-1}$

$f'(x) = -(\ln x)^{-2}(\frac{1}{x})$

$= -\frac{1}{x(\ln x)^2}$

$\frac{-\ln|x|}{x}$

-3

88

	2	5	T
100	0	1	1
90	2	3	5
80	4	2	6
70	2	6	8
60	3	0	3
59	11	9	20
	66	67.	64.2

At this Time of Year, Required students to put EACH and every Problem on the boards

This shouldn't have been so hard. I thought they were better prepared.

I'll give make up quizzes during final period as a gamble — if they take make up, that is their score. I want only serious takers!

91

	2	5	T
100	1	0	1
90	2	3	5
80	2	4	6
70	5	2	7
60	3	7	10
59	9	4	13
	1431 / 22 = 65.0	1292 / 20 = 64.6	64.45

102

6

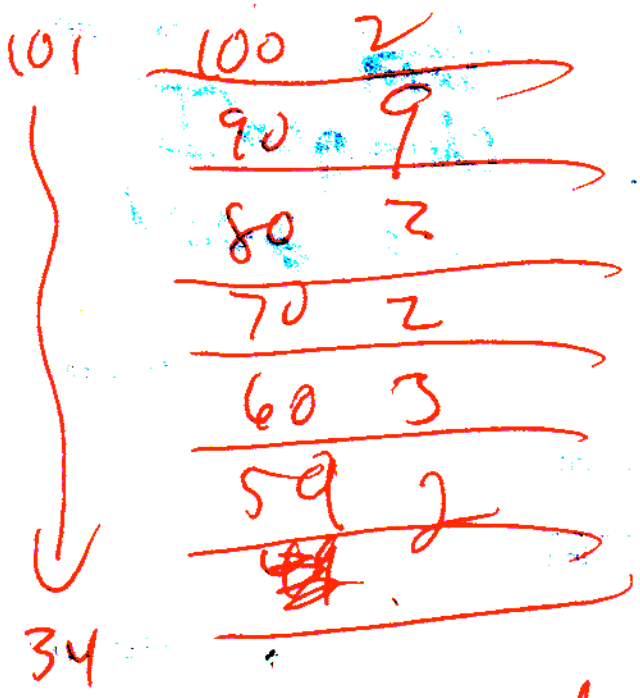
Most quit working the last week or so.

I need to emphasize problems like VI and IV and I

It looks like worksheet time maybe just get completely away from L, E etc

Show $y = \ln x$ up front then e^x as inverse fcn then a worksheet.

6/4/98



both teachers have grown up long ago

1648	1737
19	21
86.7	82.7

The main body was the same as last this exam but not at section I

good exam again.

Honors Analysis

Name ANL ch 5 6 7 8

Copy the original problem.
Convince *me* that **you** understand the concept.

Per _____ Date 98

88, 91, 98

LAST REGULAR EXAM

I Given: $f(x) = \dfrac{\ln x}{x}$ (tot 30 pts)

a) Find the domain of $f(x)$
b) Find the interval(s) where $f(x)$ is increasing.
c) Find and identify the *coordinates* of all extrema.
d) Find the *coordinates* of all points of inflection.
e) Based on sections "a" through "d", sketch $f(x)$.

~~Sin~~ changed I is 2001

II Given $f(x)$, find $f'(x)$: (5 pts ea)

a) $f(x) = \sin e^{-2x}$

b) $f(x) = \ln(\tan x)$

c) $f(x) = e^{x^e}$

d) $f(x) = \dfrac{1}{\ln x}$

III Find the integral : (5 pts ea)

a) $\displaystyle\int \frac{(2x+3)^3}{x}\,dx$

b) $\displaystyle\int e^{-kx}\,dx$

IV Simplify using the "Mumaugh" tables: (5 pts ea)

a) $\ln 40$ b) $\log_3 8$ c) $\ln (2.4)$ d) $(\log_5 2)(\log_2 25)$

V Given $e^y = x$. Find the first derivative of y with respect to x. (10 pts)

VI Find the x-intercept of the line tangent to $f(x) = \ln x$ which has slope e^2. (10 pts)

Extra Credit 5 pts

Given: $f(x) = \ln(\cos^2 x - \sin^2 x)$, find $f'\left(\dfrac{\pi}{8}\right)$.

I $f(x) = \begin{cases} 0 & \text{if } x=0 \\ x^2 \ln x & \text{if } 0<x<2 \end{cases}$

$f(e^{-\frac{1}{2}}) = \frac{1}{e}\cdot(-\frac{1}{2}) = -\frac{1}{2e}$

$f(2) = 4\ln 2$

a) decreases $0<x<e^{-\frac{1}{2}}$

b) concave dn $0<x<e^{-\frac{3}{2}}$

c) $(0,0)$ rel max dne f'n it

$(e^{-\frac{1}{2}}, -\frac{1}{2e})$ Abs min f'n it

$(2, 4\ln 2)$ Abs max inc to it

II

a) $f = \cos e^{3x}$

$f' = -3e^{3x}\sin e^{3x}$

b) $f = \ln e^{3x^2}$

$= 3x^2 \ln e$

$= 3x^2$

$f' = 6x$

c) $f = x^{e^x}$

$\ln f = e^x \ln x$

$\frac{1}{f} f' = e^x \ln x + \frac{e^x}{x}$

$f' = x^{e^x}\left[e^x \ln x + \frac{e^x}{x}\right]$

d) $f = \frac{1}{\ln x} = (\ln x)^{-1}$

$f' = -(\ln x)^{-2}\frac{1}{x}$

need $\ln x^{-1}$ instead $= -\frac{1}{x(\ln x)^2}$

$f = x^2 \ln x$

$f' = 2x\ln x + x$

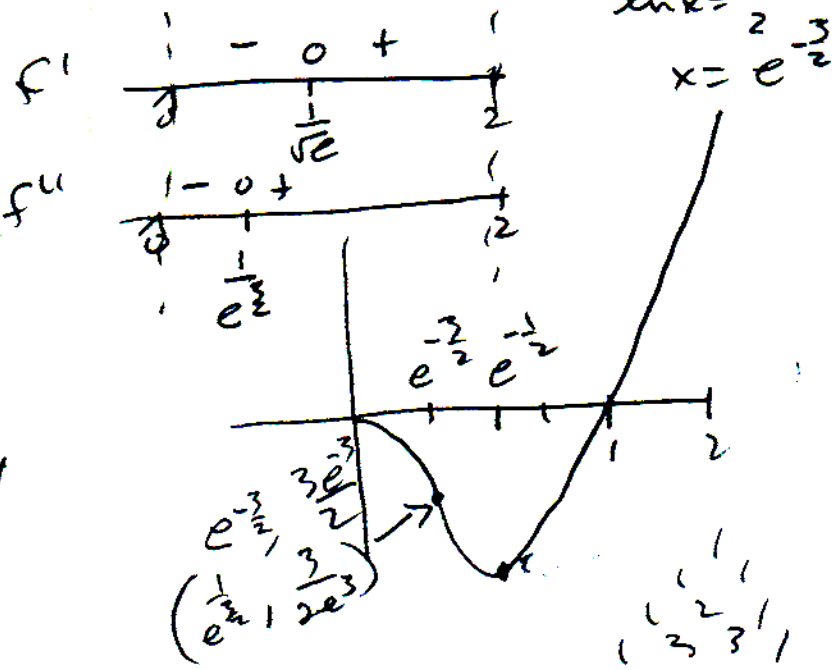
$x(2\ln x + 1) \overset{\text{set}}{=} 0$

$f'' = (2\ln x + 1) + 2 = 2\ln x + 3 \overset{\text{set}}{=} 0$

$\ln x = -\frac{3}{2}$

$x = e^{-\frac{3}{2}}$

$\ln x = -\frac{1}{2} \therefore x = e^{-\frac{1}{2}}$



III a) $\int \frac{(3-2x)^3}{x}\,dx$

$\int \frac{3^3 - 3^2(2x)\cdot 3 + 3\cdot 3(2x)^2 - (2x)^3}{x}\,dx$

$\int \frac{27}{x} - 54 + 36x - 8x^2\,dx$

$27\ln|x| - 54x + 18x^2 - \frac{8}{3}x^3 + C$

b) $\int e^{-3x}\,dx = -\frac{1}{3}e^{-3x} + C$

IV a) $\ln 40$

$3\ln 2 + \ln 5$

3a + b

$3a + c$

b) $\log_3 8$

$\frac{3\ln 2}{\ln 3}$

c) $\ln 2.4$

$\ln \frac{12}{5}$

$\ln 4 + \ln 3 - \ln 5$

$2a + b - c$

d) $(\log_5 2)(\log_2 25)$

$\frac{\ln 2}{\ln 5}\cdot\frac{2\ln 5}{\ln 2}$

2

$\frac{39}{5}$

V $f(x) = e^x \quad m = e^2$

$f' = e^x \overset{\text{set}}{=} e^2$

$x = 2$

$(2, e^2) \quad m = e^2$

$y - e^2 = e^2(x-2)$

$y = e^2x - e^2 \overset{\text{set}}{=} 0$

$x = 1$

x-intercept $(1, 0)$

VI $f(x) = \sqrt{\dfrac{(2x^2+1)^3}{(3x^2-2)^5}}$

D_f : $+ \; 0 \; - \; \text{ø} \; +$ at $-\sqrt{\frac{2}{3}}, \; 3, \; \sqrt{\frac{2}{3}}$

NO ANS and DID not use ln prop = -7

DID use ln prop = -4

$\ln y = \frac{3}{2}\ln(2x^2+1) - \frac{5}{2}\ln(3x^2-2)$

$\frac{1}{y}\frac{dy}{dx} = \frac{3}{2}\frac{4x}{2x^2+1} - \frac{5}{2}\frac{6x}{3x^2-2}$

$\times\left[\dfrac{6(3x^2-2) - 15(2x^2+1)}{(2x^2+1)(3x^2-2)}\right]$

$18x^2 - 12 - 30x^2 - 15$

$-12x^2 - 27$

$f' = \left(\dfrac{-3x(4x^2+9)}{(2x^2+1)(3x^2-2)}\right)\sqrt{\dfrac{2x^2+1}{3x^2-2}}$

No New instructions

f' : $+ \; 0 \; - \; 0 \; + \; 0 \; -$ at $-\sqrt{\frac{2}{3}}, \; 0, \; \sqrt{\frac{2}{3}}$

reject not in domain.

only valid area.

f increases when $x < -\sqrt{\frac{2}{3}}$

Extra.

$y = \dfrac{x^2}{x^2+1}$ u v

$y' = \dfrac{2x(x^2+1) - 2x(x^2)}{(x^2+1)^2}$

$= \dfrac{2x^3 - 2x - 2x^3}{(x^2+1)^2}$

$y' = \dfrac{2x}{(x^2+1)^2}$ u v

$y'' = \dfrac{2(x^2+1)^2 - 2x(2(x^2+1)(2x)}{(x^2+1)^4}$

$= \dfrac{2(x^2+1) - 8x^2}{(x^2+1)^3}$

$= \dfrac{-6x^2+2}{(x^2+1)^3}$

f'' : $+ \; 0 \; - \; \text{ø} \; +$ at $-\sqrt{\frac{1}{3}}, \; 0, \; \sqrt{\frac{1}{3}}$

$y'|_{x=\pm\sqrt{\frac{1}{2}}} = \frac{\frac{1}{2}}{\frac{4}{3}} = \frac{1}{4}$

$\left(\pm\sqrt{\frac{1}{3}}, \frac{1}{4}\right)$

$3 + 16 =$

19

93

	2	5	1
100	0	0	
90	4	2	
80	5	8	
70	2	7	
60	2	1	
59	8	3	

98

32

1475/21 = 70.2

1511/20 = 75.6

2988/41 = 72.8

94

	2	5	1
160	1	2	3
90	5	7	12
80	4	4	8
70	4	1	5
60	2	5	7
59	4	2	6

1504/20 = 75.2

1669/21 = 79.48

77.

Sections VI graded very easily. many students went on vacation about 2 weeks early!

99

	4	5	T
100	0	0	0
90	1	4	5
80	5	8	13
70	3	0	3
60	3	4	7
59	3	3	6

95

12

71, 78.7 75.5

15 18 33

97

100	1
90	3
80	6
70	3
60	2
59	10

103

33

1703

25

68.1

Honors Analysis

Name AHL ch567

Copy original problem.

Per _____ Date 99

Convince *me* that you understand the concept!

No Calculators!

Chapters 5, 6, & 7 Exam

I Given: $f(x)=\begin{cases}0 & \text{if } x=0\\ x^2\ln x & \text{if } 0<x\le 2\end{cases}$ (30 pts tot)

a) Find the interval(s) where $f(x)$ is decreasing.

b) Find the interval(s) where $f(x)$ is concave down.

c) Find, identify and verify the coordinates of *all* extrema.

d) Find the coordinates of any point(s) of inflection.

e) Draw $f(x)$. Be sure to specifically indicate any significant points.

II Given $f(x)$, find $f'(x)$: (5 pts ea)

a) $f(x)=\cos(e^{3x})$ b) $f(x)=\ln(e^{3x^2})$ c) $f(x)=x^{e^x}$ d) $f(x)=\dfrac{1}{\ln x}$

III Find the integral: (5 pts ea)

a) $\displaystyle\int\frac{(3-2x)^3}{x}\,dx$ b) $\displaystyle\int e^{-3x}\,dx$

IV Simplify using the "Mumaugh" tables: (5 ps ea)

a) $\ln 40$ b) $\log_3 8$ c) $\ln(2.4)$ d) $(\log_5 2)(\log_2 25)$

V Find the x-intercept of the line tangent to $f(x)=e^x$ which has slope e^2. (10 pts)

VI Determine the intervals where $f(x)$ is increasing: $f(x)=\sqrt{\dfrac{(2x^2+1)^3}{(3x^2-2)^3}}$ (10 pts)

Extra Credit ---------- 5 pts ----------

The curve $y=\dfrac{x^2}{x^2+1}$ has two inflection points. They are $(\pm p,q)$. Find the value of: $\dfrac{1}{p^2}+\dfrac{1}{q^2}$.

$$y = x^{x^x}$$

$$\ln y = x^x \ln x$$

$$\frac{1}{y}\frac{dy}{dx} = \ln x \left[* \right] + \frac{x^x}{x}$$

$$\frac{dy}{dx} = x^{x^x}\left[* \ln x + x^{x-1} \right]$$

$$* = \frac{dz}{dx}$$

$$z = x^x$$

$$\ln z = x \ln x$$

$$\frac{1}{z}\frac{dz}{dx} = \ln x + 1$$

$$\frac{dz}{dx} = x^x(\ln x + 1)$$

$$\frac{dy}{dx} = x^{x^x}\left[x^x(\ln x + 1) + x^{x-1} \right]$$

Honors ANALYSIS

EL QUIZZO

NO PARTIAL CREDIT

19+	+
14-18	✓
9-13	0
5-8	−
0-4	=1 pt each

NAME Chart 5, 6, 7, 8

PER _____ DATE _____

Write your answers on this paper.

I Differentiate (ie. Given y find $\frac{dy}{dx}$)

A) $y = e^{\frac{1}{x}}$ $-\frac{1}{x^2}e^{\frac{1}{x}}$

B) $y = 3x^4 + 3x + 7$ $12x^3 + 3$

C) $y = e^3$ 0

D) $y = e^{x^3+1}$ $3x^2 e^{x^3+1}$

E) $y = \left(x^{\frac{1}{2}} + e^x\right)^{\frac{1}{3}}$ $\frac{1}{3}(x^{\frac{1}{2}}+e^x)^{-\frac{2}{3}}\left[\frac{1}{2\sqrt{x}}+e^x\right]$

F) $y = x^x$ $e^{x\ln x}$ $y' = x^x[\ln x + 1]$

G) $y = e^{e^{2x}}$ $2e^{e^{2x}}\cdot e^{2x}$

H) $y = 3e^{5^2+x^2}$ $y' = 3e^{5^2+x^2}[2x]$

I) $y = \frac{1}{1+\frac{1}{x}}$ $\frac{1}{\frac{x+1}{x}} > \frac{x+1-x}{(x+1)^2}$ $\frac{1}{(x+1)^2}$

J) $y = \frac{1}{\ln x}$ $(\ln x)^{-1}$ $-(\ln x)^{-2}\cdot\frac{1}{x}$ $-\frac{1}{x(\ln x)^2}$

K) $y = \ln(2x-4)^2$ $2\ln(2x-4)$ $\frac{2\cdot 2}{2x-4}$ $\frac{2}{x-2}$

L) $y = x^{e^x}$ $\ln y = e^x \ln x$ $\frac{1}{y}\frac{dy}{dx} = e^x\ln x + \frac{e^x}{x}$ $\frac{dy}{dx} = x^{e^x}\left[e^x\ln x + \frac{e^x}{x}\right]$

II Perform the indicated operation:

A) $\int\left(3x^2+1\right)^2 dx$ $\int 9x^4+6x^2+1\,dx$ $\frac{9x^5}{5}+2x^3+x+c$

B) $\int e^{7x}+1\,dx$ $\frac{1}{7}e^{7x}+x+c$

C) $\int 10^2\,dx$ $100x+c$

D) $\int x^{-1}\,dx$ $\ln|x|+c$

III Given: $F(x) = (\ln x)(\ln x)(\ln x)$, Find $F'(x)$

$(\ln x)^3$

$f' = \frac{3(\ln x)^2}{x}$

IV Find *T H R E E* distinct functions which have the derivative :

$F'(x) = 3x^2+x-2$

1. $x^3+\frac{1}{2}x^2-2x+1$ 2. $x^3+\frac{1}{2}x^2-2x+2$ 3. $x^3+\frac{1}{2}x^2-2x$